Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Friday 27 January 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 0.6 m. The other end of the string is attached to a fixed point A. The particle is released from rest at A and comes to instantaneous rest 1.1 m below A.

Find the modulus of elasticity of the string.

(4)

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2. A particle P is moving in a straight line with simple harmonic motion. The centre of the oscillation is the fixed point C, the amplitude of the oscillation is 0.5 m and the time to complete one oscillation is $\dfrac{2\pi}{3}$ seconds. The point A is on the path of P and 0.2 m from C.

Find

(a) the magnitude and direction of the acceleration of P when it passes through A,

(3)

(b) the speed of P when it passes through A,

(2)

(c) the time P takes to move directly from C to A.

(3)

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3. A particle P is moving in a straight line. At time t seconds, P is at a distance x metres from a fixed point O on the line and is moving away from O with speed $\dfrac{10}{x+6}$ m s^{-1}.

(a) Find the acceleration of P when $x = 14$.

(4)

Given that $x = 2$ when $t = 1$,

(b) find the value of t when $x = 14$.

(6)

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4. A light elastic string AB has natural length 0.8 m and modulus of elasticity 19.6 N. The end A is attached to a fixed point. A particle of mass 0.5 kg is attached to the end B. The particle is moving with constant angular speed ω rad s^{-1} in a horizontal circle whose centre is vertically below A. The string is inclined at 60° to the vertical.

(a) Show that the extension of the string is 0.4 m.

(5)

(b) Find the value of ω.

(5)

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5. Above the Earth's surface, the magnitude of the gravitational force on a particle due to the Earth is inversely proportional to the square of the distance of the particle from the centre of the Earth. The Earth is modelled as a sphere of radius R and the acceleration due to gravity at the Earth's surface is g. A particle P of mass m is at a height x above the surface of the Earth.

(*a*) Show that the magnitude of the gravitational force acting on P is

$$\frac{mgR^2}{(R+x)^2}.$$

(3)

A rocket is fired vertically upwards from the surface of the Earth. When the rocket is at height $2R$ above the surface of the Earth its speed is $\sqrt{\left(\frac{gR}{2}\right)}$. You may assume that air resistance can be ignored and that the engine of the rocket is switched off before the rocket reaches height R.

Modelling the rocket as a particle,

(*b*) find the speed of the rocket when it was at height R above the surface of the Earth.

(9)

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6. A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is hanging in equilibrium at the point A, vertically below O, when it is set in motion with a horizontal speed $\frac{1}{2}\sqrt{(11gl)}$. When the string has turned through an angle θ and the string is still taut, the tension in the string is T.

(*a*) Show that $T = 3mg\left(\cos\theta + \frac{1}{4}\right)$.

(8)

At the instant when P reaches the point B, the string becomes slack.

Find

(*b*) the speed of P at B,

(3)

(*c*) the maximum height above B reached by P before it starts to fall.

(4)

7.

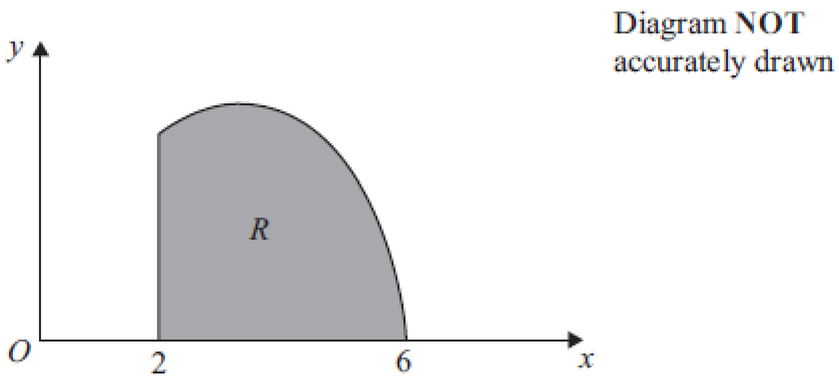


Figure 1

The shaded region R is bounded by the curve with equation $y = \frac{1}{2}x(6 - x)$, the x-axis and the line $x = 2$, as shown in Figure 1. The unit of length on both axes is 1 cm. A uniform solid P is formed by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of P is, to 3 significant figures, 1.42 cm from its plane face.

(9)

The uniform solid P is placed with its plane face on an inclined plane which makes an angle θ with the horizontal. Given that the plane is sufficiently rough to prevent P from sliding and that P is on the point of toppling when $\theta = \alpha$,

(b) find the angle α.

(4)

Given instead that P is on the point of sliding down the plane when $\theta = \beta$ and that the coefficient of friction between P and the plane is 0.3,

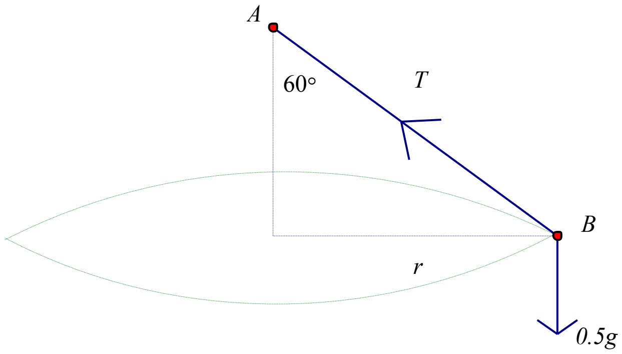
(c) find the angle β.

(3)

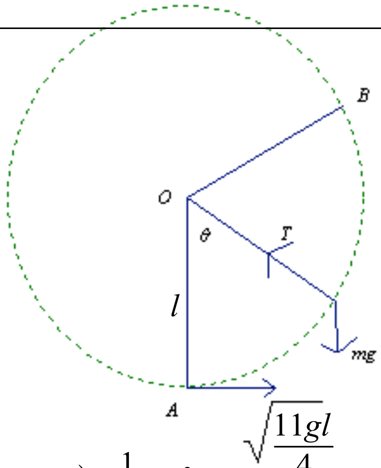
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	EPE $= \dfrac{\lambda \times 0.5^2}{1.2}$	B1
	GPE lost = EPE gained	M1 (used)
	$0.8 \times 9.8 \times 1.1 = \dfrac{\lambda \times 0.5^2}{1.2}$	A1ft
	$\lambda = 41.4$ N or 41 N	A1
		(4 marks)
2.		
(a)	$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3}, \quad \omega = 3$	B1
	$\lvert a \rvert = \omega^2 x = 9 \times 0.2 = 1.8\,\text{ms}^{-2}$ towards C	M1 A1
		(3)
(b)	$v^2 = \omega^2\left(a^2 - x^2\right) = 9(0.25 - 0.04) = 1.89$	M1
	$v = 1.37\ \text{ms}^{-1}$	A1
		(2)
(c)	$x = 0.5\sin 3t = 0.2$	M1 A1ft
	$t = \dfrac{1}{3}\sin^{-1} 0.4 \approx 0.137\,\text{s}$	A1
		(3)
		(8 marks)
3.		
(a)	$a = v\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{10}{x+6} \times \dfrac{-10}{(x+6)^2},\ = \dfrac{-100}{(x+6)^3}$	M1 M1, A1
	$= \dfrac{-100}{(14+6)^3} = -\dfrac{1}{80}\,\text{ms}^{-2}$	A1
		(4)
(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{10}{x+6} \Rightarrow \int x + 6\mathrm{d}x = \int 10\mathrm{d}t$	M1 M1
	$\left[\dfrac{x^2}{2} + 6x\right]_2^{14} = \left[10t\right]_1^T$	M1 A1
	$\dfrac{196}{2} + 6 \times 14 - 2 - 12 = 10T - 10$	M1
	$178 = T \qquad T = 17.8\text{(s)}$	A1
		(6)
		(10 marks)

Question Number	Scheme	Marks
4.		
(a)		
	$\uparrow\ T\cos 60° = 0.5g$, $T = g$ (1)	M1, A1
	Extension in the string $= x$, $T = \dfrac{\lambda x}{a} = \dfrac{19.6x}{0.8}$	B1
	Using (1), $g = 24.5x$, $x = 0.4$ m *	M1, A1
		(5)
(b)	$\rightarrow\ T\sin 60° = 0.5 \times r \times \omega^2$ (2)	M1 A1
	Using (2) $g\sin 60° = 0.5 \times (0.8 + 0.4)\sin 60°\omega^2$	M1 A1
	$\omega^2 = \dfrac{2g}{1.2}$, $\omega = \sqrt{\dfrac{5g}{3}}$ (4.04 or 4.0)	A1
		(5) **10**

Question Number	Scheme	Marks
5. **(a)**	Distance of P from the centre of the Earth = $R + x$	
	$F = \dfrac{k}{(R+x)^2}$	
	$x = 0$, $F = mg$, $k = mg(R)^2$	M1 A1
	$F = \dfrac{mgR^2}{(R+x)^2}$ *	A1
		(3)
(b)	$F = ma$, $-\dfrac{gR^2}{(R+x)^2} = v\dfrac{dv}{dx}$	M1 A1
	$\displaystyle\int_V^{\sqrt{\frac{gR}{2}}} v\,dv = \int_R^{2R} -\frac{gR^2}{(R+x)^2}dx$	M1 A1
	$\left[\frac{1}{2}v^2\right]_V^{\sqrt{\frac{gR}{2}}} = \left[\frac{gR^2}{R+x}\right]_R^{2R}$	M1 A1
	$\frac{1}{2}\times\frac{gR}{2} - \frac{1}{2}V^2 = \frac{gR^2}{3R} - \frac{gR^2}{2R} = -\frac{gR}{6}$	M1
	$\frac{V^2}{2} = \frac{gR}{4} + \frac{gR}{6} = \frac{5gR}{12}$ $V^2 = \frac{5gR}{6}$, $V = \sqrt{\frac{5gR}{6}}$	A1, A1
		(9) **(12 marks)**

Question Number	Scheme	Marks
6. (a)		
	GPE gained = $mgl(1-\cos\theta)$	
	Conservation of energy: $\frac{1}{2}m\frac{11gl}{4} = mgl(1-\cos\theta)+\frac{1}{2}mv^2$	M1A1 A1
	$v^2 = gl\left(\frac{11}{4}-2+2\cos\theta\right) = gl\left(\frac{3}{4}+2\cos\theta\right)$	
	Resolving towards the centre of the circle:	M1
	$T - mg\cos\theta = \frac{mv^2}{l}$	A1 A1
	$T - mg\cos\theta = mg\left(\frac{3}{4}+2\cos\theta\right)$	M1
	$T = mg\left(\frac{3}{4}+3\cos\theta\right) = 3mg\left(\cos\theta+\frac{1}{4}\right)$ *	A1
		(8)
(b)	$T = 0 \Rightarrow \cos\theta = -\frac{1}{4}$	M1
	$v^2 = gl\left(\frac{3}{4}+2\cos\theta\right) = \frac{gl}{4}, \quad v = \sqrt{\frac{gl}{4}}$	M1, A1
		(3)
(c)	Horizontal component of velocity at B	
	$= \sqrt{\frac{gl}{4}} \times \cos(180-\theta) = \frac{1}{4}\sqrt{\frac{gl}{4}}$	B1ft
	Extra height $h \Rightarrow mgh + \frac{1}{2}m\frac{gl}{64} = \frac{1}{2}m\frac{gl}{4}$	M1 A1
	$h = \left(\frac{1}{8}-\frac{1}{128}\right)l = \frac{15l}{128}$ $(0.117l)$	A1
	OR: Using $h = \frac{v^2\sin^2\theta}{2g} = \frac{\frac{gl}{4}\times\frac{15}{16}}{2g} = \frac{15l}{128}$	(4)
	OR: Using $v^2 = u^2 + 2as$, $0 = \frac{15gl}{64} - 2gh$, $h = \frac{15l}{128}$	
		(15 marks)

Question Number	Scheme	Marks
7.		
(a)	$\int \pi y^2 \mathrm{d}x = \frac{\pi}{4}\int x^2(6-x)^2 \mathrm{d}x = \frac{\pi}{4}\int 36x^2 - 12x^3 + x^4 \mathrm{d}x$	M1 A1
	$= \frac{\pi}{4}\left[12x^3 - 3x^4 + \frac{x^5}{5}\right]_2^6 = \frac{\pi}{4} \times \frac{1024}{5}$	M1
	(160.8….)	
	$\int \pi y^2 x \mathrm{d}x = \frac{\pi}{4}\int x^3(6-x)^2 \mathrm{d}x = \frac{\pi}{4}\int 36x^3 - 12x^4 + x^5 \mathrm{d}x$	M1 A1
	$= \frac{\pi}{4}\left[9x^4 - \frac{12}{5}x^5 + \frac{1}{6}x^6\right]_2^6 = \frac{\pi}{4} \times \frac{10496}{15}$	M1
	(549.5……)	
	$\Rightarrow \bar{x} = \frac{10496}{15} \times \frac{5}{1024} = 3.416......$	M1 A1
	Required distance ≈ 3.42 – 2 = 1.42(cm) *	A1
		(9)
(b)	Base has radius $\frac{1}{2} \times 2 \times 4 = 4\,\text{cm}$	B1
	About to topple $\Rightarrow \tan\alpha = \frac{4}{1.42}$	M1 A1
	$\alpha \approx 70.5°$	A1
		(4)
(c)	Parallel to slope: $F = mg\sin\beta$	
	Perpendicular to the slope: $R = mg\cos\beta$	M1 A1
	About to slip: $F = \mu R$	
	$\tan\beta = \mu = 0.3, \quad \beta \approx 16.7°$	A1
		(3)
		(16 marks)